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# LONGEVITY RISK IN LIVING BENEFITS

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# Longevity risk in living benefits*

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**Abstract**

Uncertainty in mortality and disability trends, from which the longevity risk in living benefits arises, is discussed. The financial impact of longevity risk on life annuities, sickness benefits for the elderly and long-term care covers is then analysed, focussing in particular solvency requirements and reinsurance arrangements. Finally, special attention is devoted to mortality guarantees and flexibility in the structure of life annuities, looking at annuities in the context of overall post-retirement income planning.

**Keywords**

Mortality trends, Longevity risk, Life annuities, Long Term Care insurance, Sickness benefits, Post-retirement income, Solvency, Reinsurance.

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# 1 Introduction

Recent trends in mortality lead to the use of projected survival models when pricing and reserving for life annuities and other long-term living benefits. Several projection models have been proposed and are actually used in actuarial practice. However, the future mortality trend is random and hence, whatever kind of model is adopted, systematic deviations from the forecasted mortality may take place. Then, a "model" (or a "parameter") risk arises, which is clearly a non-pooling risk. Changes in the mortality pattern refer to both young and old ages. When the random mortality trend at old ages is concerned, it is usually referred to as "longevity risk".

Life annuities are probably the most important insurance product concerned by the longevity risk. Nevertheless, this risk should be carefully considered also when dealing with other insurance covers, especially within the area of health insurance. In particular, the longevity risk affects sickness benefits for the elderly (for example, post-retirement sickness benefits) and long-term care (LTC) annuities. A moving scenario in which both future mortality and future senescent disability are random, constitutes the appropriate context for pricing and reserving for LTC products.

The impact of the longevity risk on living benefits must be carefully faced. Reinsurance policies and capital allocation can provide appropriate tools to face this risk. Nevertheless, also the problem of "locating" the longevity risk via a possible sharing between, say, the annuity provider and the annuitant should be carefully considered.

The present paper does not contain any original idea, the presentation being mostly based on research work recently performed in the Dpt. of Applied Mathematics of the University of Trieste. The paper simply aims at providing an introduction to some important issues concerning the longevity risk in the area of the insurances of the person. Albeit the longevity risk constitutes a topic of dramatic interest in the field of actuarial mathematics, the presentation follows an informal style to facilitate understanding by non-actuarial readers.

The paper is organized as follows. Section 2 introduces a moving mortality scenario. Mortality trends and the consequent need for appropriate projections are illustrated. Mortality risks are then introduced, with particular emphasis for longevity risk.

Section 3 generalizes some ideas, considering future combined mortality and disability scenarios, hence paving the way for addressing longevity risk in LTC insurance products.

The impact of longevity risk on living benefits is discussed in Section 4. Following the description of a probabilistic approach to measure the impact of the risk itself, some numerical examples are provided, concerning various types of living benefits.

In Section 5 some ideas concerning the structure of life annuities are discussed. In particular, some issues concerning flexibility features of the life annuity product, aiming at adding value to the product itself, are dealt with, looking at annuities in the context of overall post-retirement income planning. The concept of "mortality guarantee level" is then introduced, in order to express the sharing of the mortality risks between the insurer and the annuitant.

Some final remarks, presented in Section 6, conclude the paper.

# 2 A moving mortality scenario

This Section deals with mortality trends and the need for mortality projections. Finally, the longevity risk is addressed. Recent trends in mortality are described for example by MacDonald et al. (1998) and Rüttermann (1999).

Several projection models have been proposed and are actually used in actuarial practice; the reader can refer for example to Benjamin and Soliman (1993), CMIR (1990), CMIR (1999). Lee (2000) describes the Lee-Carter projection method, which allows for randomness in future mortality trends. The seminal paper by Cramer and Wold (1935) provides us with one of the first examples of a rigorous approach to mortality projections; the reader can refer to this paper and the references therein for information about the earliest projection models.

The impact of the longevity risk on life insurance business is analysed by Olivieri (2001), where future mortality trends at young ages and old ages as well are considered. The paper by Riemer-Hommel and Trauth (2000) deals with longevity risk according to a risk management perspective.

A number of papers deal with the longevity risk in living benefits; some of them are referred to at the beginning of Section 4.

## 2.1 Trends

In many countries, mortality experience over the last decades shows some aspects affecting the shape of curves representing the mortality as a function of the attained age. Figures 2.1 and 2.2 illustrate the moving mortality scenario referring to the Italian male population, in terms of survival functions $l_x$ (number of survivors as a function of the attained age $x$) and curves of deaths (number $d_x$ of people dying as a function of $x$). Survival functions and curves of deaths relate to various cross-sectional mortality experiences.

Obviously, experienced trends also affect the behaviour of other quantities expressing the mortality pattern, such as the life expectancy and the mortality rates. Figure 2.3 illustrates the behaviour of the life expectancy at birth, for males and females in the Italian population. In Figure 2.4, referring to Italian males, the behaviour of the life expectancy at birth, the life expectancy at age 65 and the mode of the curve of deaths (i.e. the most probable dying age) are compared.

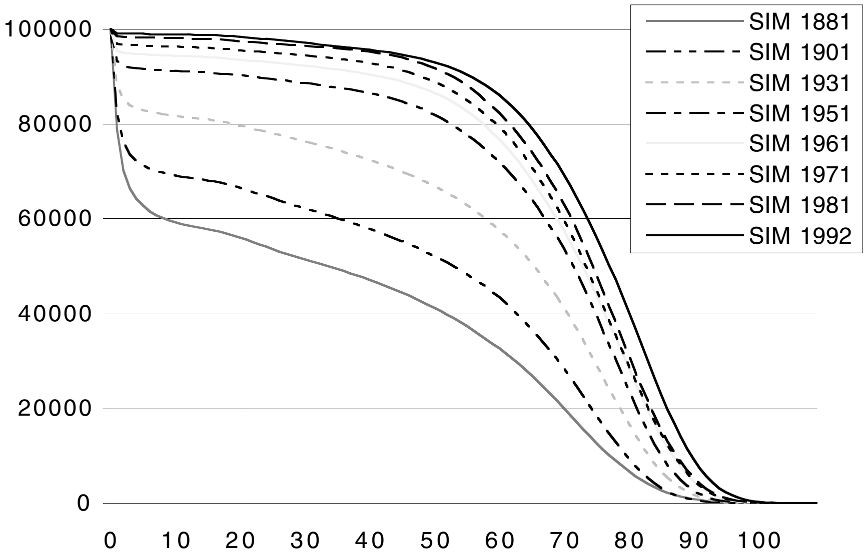


FIGURE 2.1 - *Survival functions* $l_x$ *(Italian male population)*

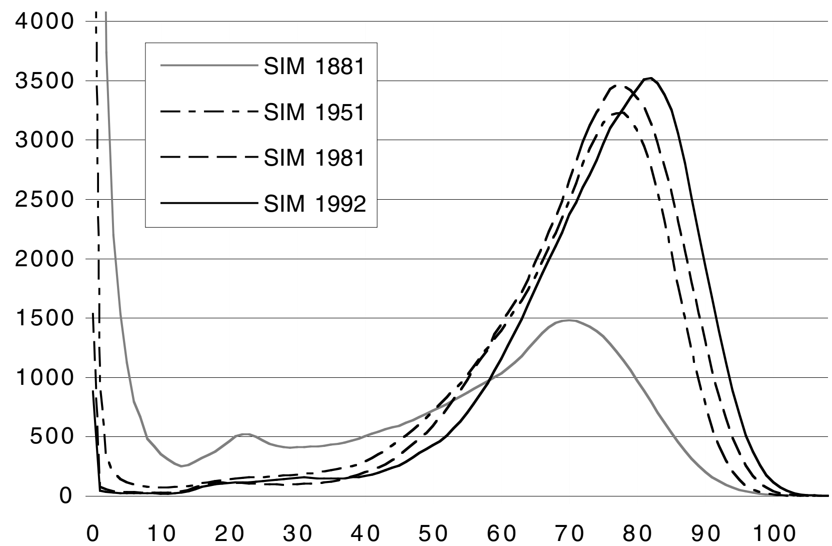


FIGURE 2.2 - *Curves of deaths* $d_x$ *(Italian male population)*

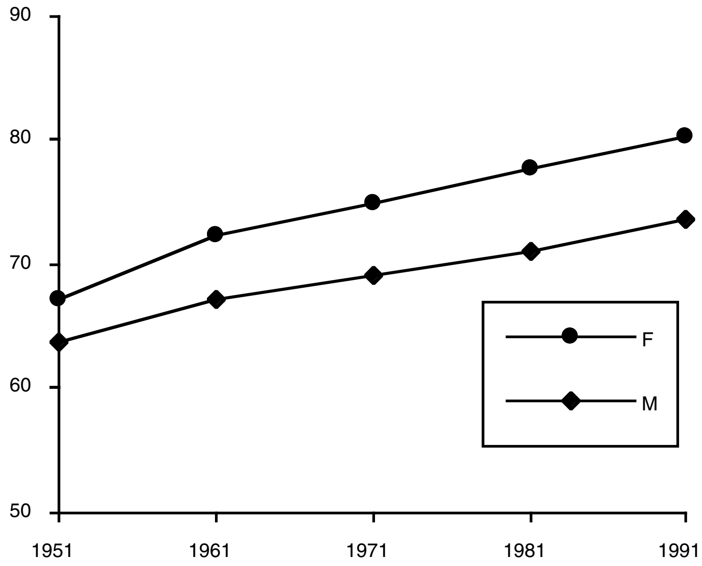


FIGURE 2.3 - *Life expectancy at birth for males [M] and females [F] (Italian population)*

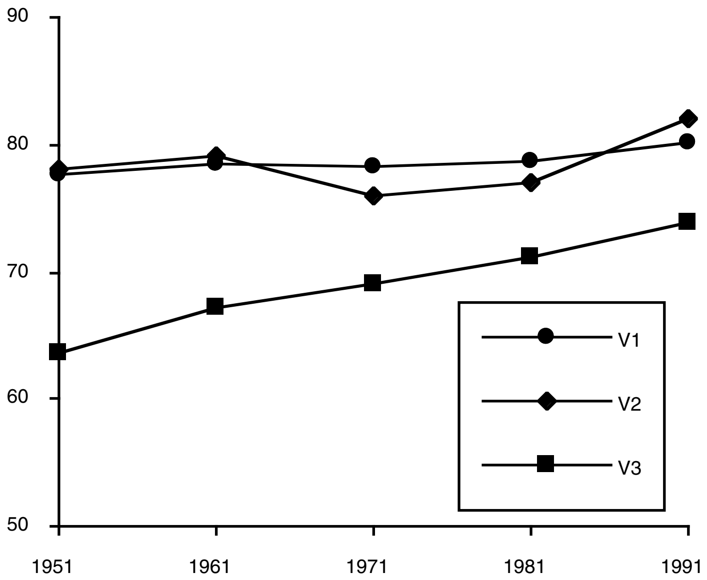


FIGURE 2.4 - *Life expectancy at age 65 [V1], Lexis point [V2], life expectancy at birth [V3] (Italian male population)*

Finally, Figures 2.5 and 2.6 concern the behaviour of mortality rates. In Figure 2.4 mortality rates $q_x$ referring to various mortality tables are plotted against the age $x$, while Figure 2.6 shows the so-called mortality profile at age 70 in relative terms, i.e. the mortality rates $q_{70}(y)$ in various calendar years $y$ divided by the mortality rate $q_{70}(1881)$ referring to the oldest table considered.

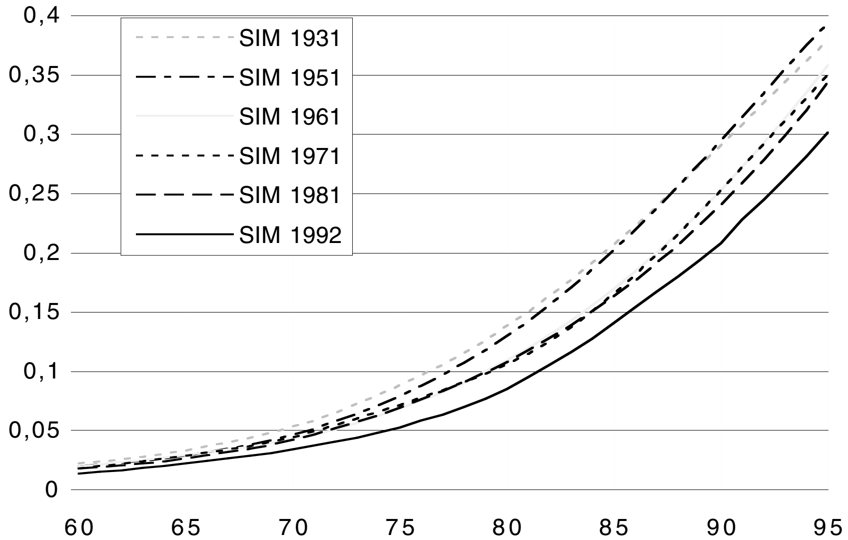


FIGURE 2.5 - *Mortality rates* $q_x$*;* $60 \le x \le 95$ *(Italian male population)*

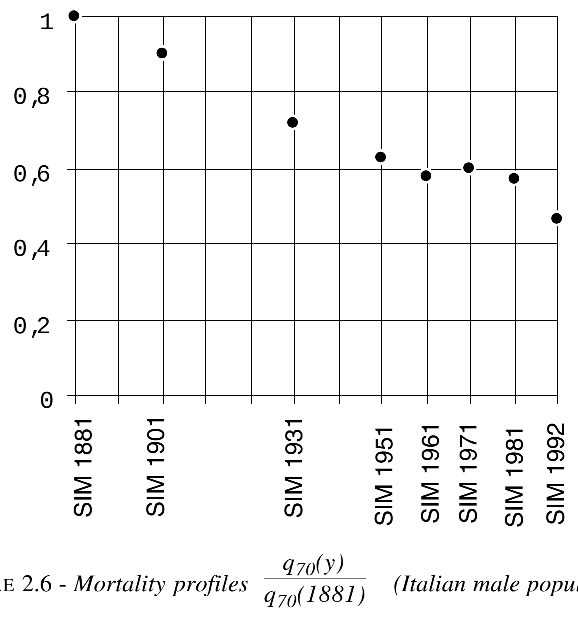


FIGURE 2.6 - *Mortality profiles* $\frac{q_{70}(y)}{q_{70}(1881)}$ *(Italian male population)*

Results are self-evident. In particular the following aspects can be pointed out: an increase in the life expectancy (at birth as well as at old ages), an overall increase in the most probable age of death, a decrease in mortality rates in particular at adult and old ages.

Turning back to the shape of the survival function and the curve of deaths, the following aspects of mortality in many countries can be singled out (for example, see Olivieri (2001)):

(a) an increasing concentration of deaths around the mode (at old ages) of the curve of deaths is evident; so the survival function moves towards a rectangular shape, whence the term "rectangularization" to denote this aspect (see Figure 2.7);

(b) the mode of the curve of deaths (which, owing to the rectangularization, tends to coincide with the maximum age $\omega$) moves towards very old ages; this aspect is called "expansion" of the survival function (see Figure 2.8);

(c) higher levels and a larger dispersion of accidental deaths at young ages (the so-called young mortality hump) have been more recently observed.

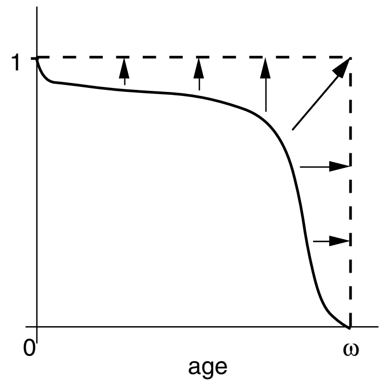


FIGURE 2.7 - *Rectangularization*

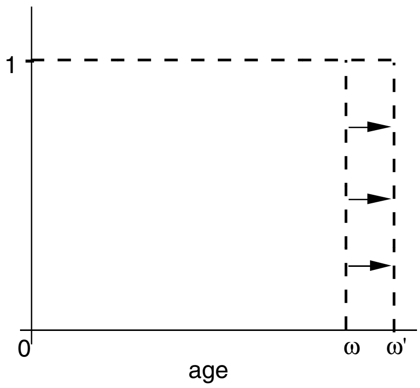


FIGURE 2.8 - *Expansion*

In Section 2.3 the importance of aspects (a) and (b) in the context of mortality risk analysis will be stressed.

## 2.2 Projections

Recent trends in mortality lead to the use of projected survival models for several actuarial purposes, e.g. for pricing and reserving for life annuities and other long-term living benefits.

When projecting mortality, the basic idea is to express the mortality itself as a function of the (future) calendar year $y$. If a single-figure representation of mortality is concerned, a projected mortality model is a real-valued function $\Psi(y)$. For example, the expected lifetime for a newborn, denoted by $e_0$ in a non-projected context, is represented by $e_0(y)$, a function of the calendar year $y$, when the future mortality trend is allowed for.

In actuarial calculations, the expression of mortality as a function of the age is needed. Then, in a projected context mortality at any age $x$ must be considered as a function of the calendar year $y$. Hence, in a rather general setting, a projected mortality model is a function $\Phi(x,y)$, which may be a real-valued or a vector-valued function. The function (and, in particular, its parameters) is constructed by applying appropriate statistical procedures to past mortality experience.

In concrete terms, a real-valued function $\Phi$ may represent mortality rates, mortality odds, a force of mortality, a survival function, some transform of the survival function, etc.

Projection models often consist in straight extrapolation procedures of the mortality profile observed in the past (see Figure 2.9). It is worth noting that inconsistencies may emerge as a result of the extrapolations; for example we may find, for some calendar year $y$, $q_{x'}(y) > q_{x''}(y)$ with $x' < x''$, even at old ages. Hence, appropriate adjustments may be required.

Conversely, projection procedures based on mortality laws (say Gompertz, Makeham, Weibull, Heligman-Pollard, etc.) allow us to express the main features of the evolving scenario, such as the rectangularization and the expansion.

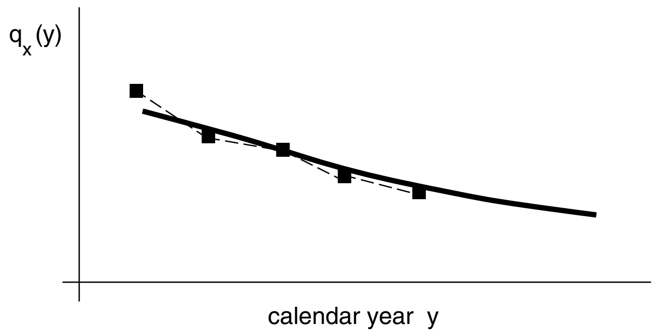


FIGURE 2.9 - *Extrapolation of the mortality profile*

## 2.3 Mortality risks. The longevity risk

Figures 2.10, 2.11 show projected mortality rates at a given age $x$ (the continuous line) and two sets of possible future mortality experience (the dots). Deviations from the projected mortality rates in Figure 2.10 can be sensibly explained in terms of random fluctuations of the outcomes (the observed mortality rates) around the relevant expected values (the projected mortality rates).

The risk of random fluctuations is a well-known type of risk in the insurance business, in both the life and the non-life insurance areas. It is often named "process risk". Fundamental results in risk theory state that the severity of this risk (conveniently assessed) decreases as the portfolio size increases. For this reason, random fluctuation risk is called a "pooling risk".

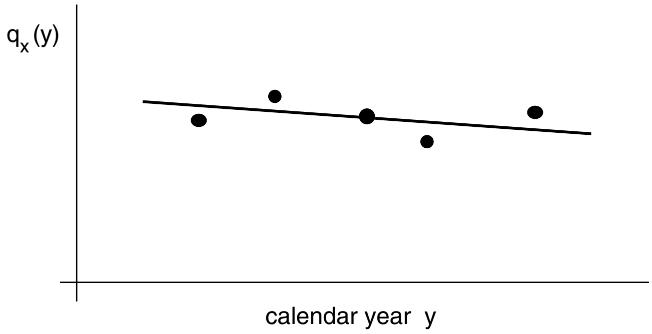


FIGURE 2.10 - *Experienced mortality: random fluctuations*

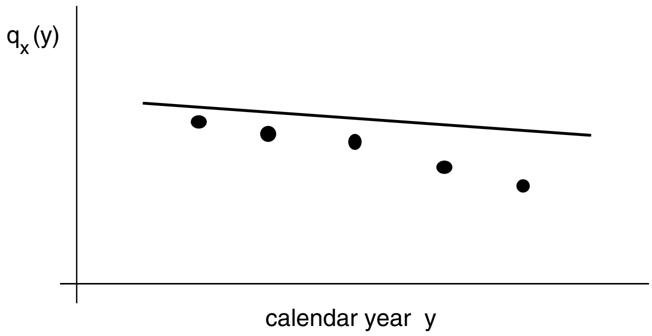


FIGURE 2.11 - *Experienced mortality: systematic deviations*

The experienced profile depicted in Figure 2.11 can be hardly attributed to random fluctuations only. Much more likely, this profile can be explained as the result of an actual mortality trend different from the forecasted one. So, systematic deviations arise. The risk of systematic deviations can be thought of as a "model risk" or "parameter risk", referring to the model used for projecting mortality and the relevant parameters.

The risk of systematic deviations cannot be hedged increasing the portfolio size. On the contrary, its financial impact increases as the portfolio size increases, since deviations concern all the insureds in the same direction. For this reason, the systematic deviation risk is called a "non-pooling risk".

It is worth stressing that the future mortality trend is obviously random and hence, whatever kind of projection procedure is adopted, systematic deviations from the forecasted mortality may occur.

Depending on the statistical model adopted in analysing past data and forecasting mortality, in some cases the assessment of uncertainty in future mortality trends constitutes an output of the statistical model itself. For simplicity, let us consider the projection of a single-figure representation of mortality, such as the life expectancy at age 0. For any (future) time $y$, the projected value $e_0(y)$ can be considered a point estimate, around which an interval estimate provides a probabilistic insight about possible future trends (see Figure 2.12, in which the shaded region represents possible evolutions of the life expectancy at birth).

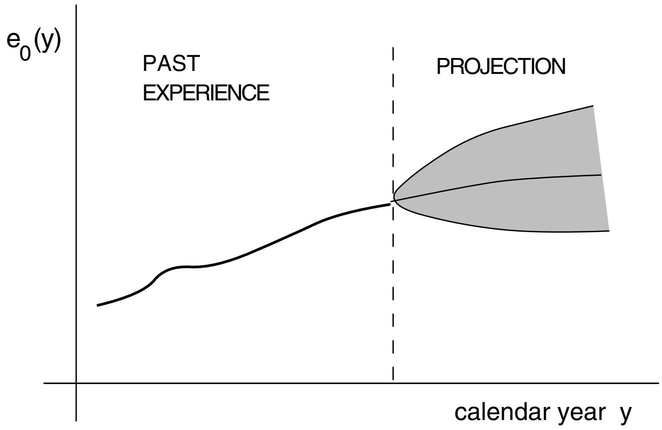


FIGURE 2.12 - *Interval estimates for forecasted life expectancy*

In general terms, thus disregarding the possibility of finding a risk assessment arising from the statistical procedures adopted, in what follows we focus on possible systematic deviations from the (point) estimation of future mortality. In particular, we are interested in the consequences of this risk on valuations concerning living benefits. Restricting our attention to trends of the mortality pattern at old ages only, we will refer to this risk as the "longevity risk".

# 3 Future mortality and disability scenarios

Long Term Care (LTC) is care required in relation to chronic (or long-lasting) bad health conditions. LTC insurance provides income support for the insured, who needs nursing and/or medical care, in the form either of a forfeiture annuity benefit or nursing and medical expense refunding. Given the type of claim covered, LTC insurance has a lifetime duration. In what follows we focus on forfeiture annuity benefits, which represent the most common type of LTC benefit.

When LTC annuities are concerned, combined mortality and disability scenarios constitute the natural framework for actuarial evaluations. First, it is evident that this type of living benefits is affected by the longevity risk pertaining to both healthy and disabled people. Moreover, the uncertainty in future disability inception rates (or disability prevalence rates) for the elderly should be carefully considered.

For the structure of LTC products and the relevant actuarial aspects, the reader can refer to Haberman and Pitacco (1999). Trends in disability are dealt with by Mayhew (2001). The paper by Ferri and Olivieri (2000) addresses longevity risk in LTC covers, allowing for randomness in both mortality and disability trends. For more information about combined mortality and disability future scenarios, the reader should refer to the extensive set of references in the two papers mentioned above.

## 3.1 Combining mortality and senescent disability

As far as future trends are concerned, three main theories have been formulated about the combined evolution of scenescent disability and mortality. The relevant scenarios are illustrated in Figure 3.1, in terms of expected lifetimes as functions of the calendar year.

(i) "Compression theory": chronic degenerative diseases will be postponed until the latest years of life because of medical advances. Assuming there is a maximum age, these improvements will result in a compression of the period of disability.

(ii) "Pandemic theory": the reduction in mortality rates is not accompanied by a decrease of disability rates; hence, the number of disabled people will increase steadily.

(iii) "Equilibrium theory": most of the changes in mortality are related to specific pathologies. The onset of chronic degenerative diseases and disability will be postponed and the time of death as well.

The scenarios depicted by the above mentioned theories produce different consequences for LTC insurers, as far as in-force LTC portfolios are concerned. In particular, compression theory suggests optimistic views, whilst pandemic theory pessimistic ones. The dramatic differences among such theories imply a high level of uncertainty about the evolution of senescent disability. The adoption of projected tables for the evaluation of insured benefits seems necessary; however, since the three theories imply quite different scenarios, the mentioned uncertainty should be included in the evaluation model.

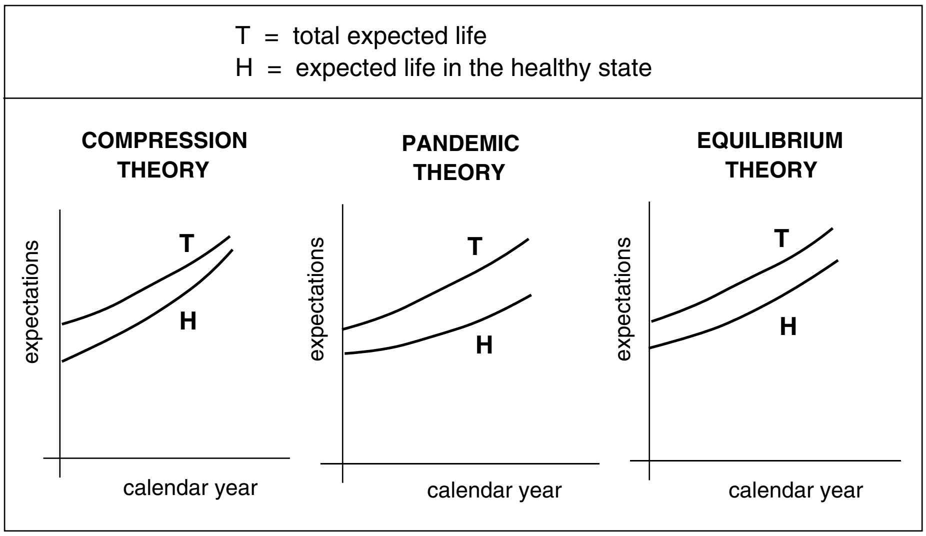


FIGURE 3.1 - *Combined mortality and senescent disability scenarios*

### 3.2 The demand for LTC insurance

LTC insurers are concerned by future combined mortality and senescent disability trends also when future acquisition of LTC policies is considered. Actually, we can imagine that trends following the compression theory would cause, at an individual level, a decreasing need in LTC and hence a decreasing demand for LTC insurance products, whilst trends following the equilibrium theory would cause a roughly constant need and demand; on the contrary, pandemic trends would imply an increasing need and demand for LTC products.

However, at a collective level, the demand depends on the size of the elderly population. Hence, assuming an increasing elderly population (as supported by population projections also taking into account low fertility levels), we find that trends following the compression theory would cause a roughly stable demand for LTC products, while equilibrium trends would imply an increasing demand; finally, a dramatically increasing demand would be the consequence of trends following the pandemic theory.

## 4 Assessing and facing the longevity risk

The following problems are dealt with in this Section: (a) how to express the longevity risk; (b) how to assess it; (c) how to face it.

Basic ideas underlying the approach to problem (a) are rather general and can be implemented whatever type of living benefits is concerned. Nevertheless, for the sake of simplicity we shall restrict our attention to benefits only depending on the residual lifetime, viz. straight life annuities.

Problems (b) and (c) can be dealt with only addressing specific categories of insurance products. So, we shall separately consider life annuities, LTC insurance and sickness benefits for the elderly.

Ideas and results presented in this Section come from the following papers; a more formal presentation can be found in Pitacco (2001). The impact of longevity risk on life annuities has been dealt with by Marocco and Pitacco (1998), where reinsurance arrangements facing this risk are also discussed. Olivieri (2001) considers future mortality trends at young ages and old ages as well, and suggests an assessment of the impact of systematic deviations on term insurance and life annuities portfolios. The longevity risk in life annuities portfolio and the relevant solvency requirements are dealt with by Olivieri and Pitacco (2000), also allowing for investment risk. A joint analysis of financial and mortality risks has been proposed by Coppola, Di Lorenzo and Sibillo (2000). The paper by Olivieri and Pitacco (2002) deals with Bayesian inference on mortality improvements.

The paper by Ferri and Olivieri (2000) concerns LTC benefits in a moving scenario in which both future mortality and future senescent disability are random. Olivieri and Pitacco (2001a) deal with the enhanced pension, i.e. a particular LTC product; the costs for financing a proper solvency reserve, tailored to the characteristics and magnitude of the risks, and a stop-loss reinsurance arrangement are analysed.

The longevity risk affecting sickness benefits for the elderly (for example, post-retirement sickness benefits) is analysed by Olivieri and Pitacco (1999). Finally, Biffis and Olivieri (2002), referring to pension schemes, show how packaging benefits of various types can reduce the overall impact of longevity risk.

## 4.1 Future scenarios: deterministic vs probabilistic approach

The longevity risk arises from uncertainty in future mortality (and possibly disability) trends. So, when focussing our attention on the impact of this risk, first we have to choose a set of sensible future scenarios, e.g. a set (or "space") of survival functions. The space of survival functions can be a discrete set (and in particular a finite one), or a continuous one (in this case survival functions should be assigned via a mathematical model, whose parameters take values in given intervals).

The choice of a set of future scenarios allows us to check the consequences of the longevity risk, for example in terms of expected numbers of surviving annuitants or expected payments by the insurer, adopting a "deterministic" approach. According to this approach, different reasonable scenarios (e.g. an "optimistic" scenario, a "medium" one, etc.) are considered and the relevant calculations are performed. Such an approach is usually called "scenario testing". It should be stressed that the scenario testing allows for the random fluctuation risk, whereas it provides just a rough information about the longevity risk, typically through ranges for some results.

Conversely, a probabilistic approach consists in considering each scenario as a possible outcome to which some "measure" is assigned. So, a second step in the modelling process is required, viz. the expression of our degree of belief in the various scenarios; hence, a probability distribution over the set of scenarios must be assigned.

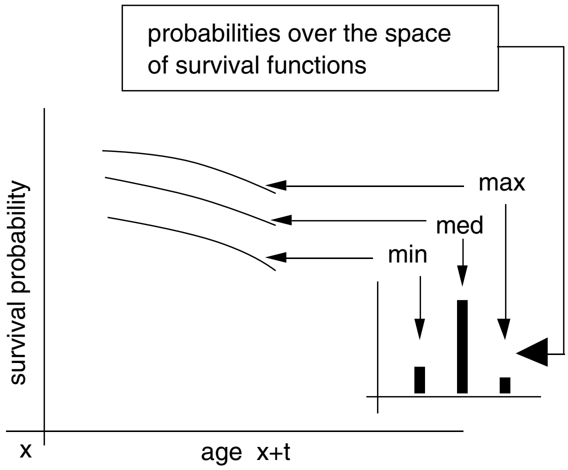


FIGURE 4.1 - *The uncertainty in mortality trends: a discrete approach*

If the scenarios constitute a finite set, a probability should be assigned to each scenario, i.e. to each survival function (see Figure 4.1). On the contrary, when a continuous set is concerned, a probability density function can express our degree of belief, referring for example to some parameter of the survival function. The choice of the density function can be driven by the values assumed by some projected survival functions (see Figure 4.2, where three projected survival functions are used as a starting point).

In what follows we will use a finite set of scenarios, adopting a probabilistic approach. So a set of probabilities will be assigned, each probability representing our degree of belief in the corresponding scenario.

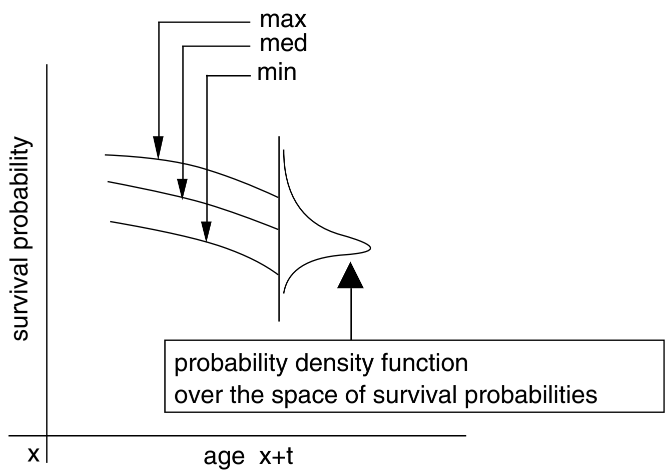


FIGURE 4.2 - *The uncertainty in mortality trends: a continuous approach*

## 4.2 Life annuities

It is worth noting that a projected survival function allowing for a high degree of rectangularization leads to a mortality risk lower than a function with a smaller degree of rectangularization. Actually, a highly rectangularized survival function implies a strong concentration of deaths around the Lexis point and hence a lower variance of the random lifetime. This results in a reduced risk of random fluctuations in mortality, and then in a reduced mortality risk in a portfolio of life annuities.

However the degree of the expansion phenomenon is unknown, whence the future location of the Lexis point is random. Then, the insurer must face the risk coming from the unknown maximum probable age of death, and hence the risk of systematic deviations.

Several quantities can be used as "functions" in order to assess the effect of mortality risks, and in particular to split the overall risk into its components, the random fluctuation risk and the systematic deviation risk. Referring to a portfolio of single premium life annuities, we can in particular focus on the following quantities:

- the variance or the standard deviation of the random present values of the insurer's future payments;
- the risk index, i.e. the ratio between the standard deviation of the random present value of the insurer's future payments and its expected value;
- the percentiles of the probability distribution of the random present value of the insurer's future payments;

- the required solvency margin, conveniently defined in order to allow for the specific risk profile of the insurer.

Here we focus on the required solvency margin, following the approach adopted by Oliveri and Pitacco (2000).

Let us assume three projected mortality scenarios, each described by the Heligman-Pollard law, with various degrees of rectangularization and expansion. In Figure 4.3 the three scenarios, labelled [min], [med] and [max] respectively, are represented in terms of the curve of deaths, and compared with a scenario, labelled [C], assumed to represent the result of recent cross-sectional observations. The following probabilities are attributed to the projected scenarios:

$$\rho^{[\min]} = 0.2;\ \ \rho^{[\mathrm{med}]} = 0.6;\ \ \rho^{[\max]} = 0.2.$$

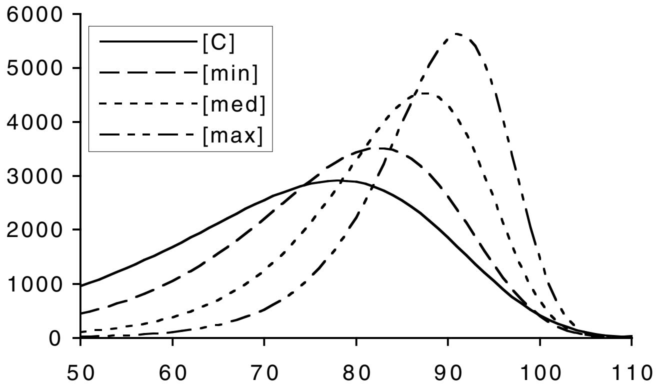


FIGURE 4.3 - *Curves of deaths following the Heligman-Pollard law*

A portfolio consisting of a single cohort of annuitants has been considered. All annuitants are aged 65. The annual amount of the annuity is the same for all annuitants. Single premiums are calculated with the mortality assumption provided by the scenario [med]; the same scenario is used to evaluate the portfolio reserve. No investment risk has been considered. The following solvency criterium has been adopted.

Let $t = 0$ denotes the time of solvency ascertainment. The probability that in all years, i.e. at times $t = 1, 2, \ldots$, the assets allocated to the portfolio are greater or equal to the liabilities of the portfolio itself must assume an assigned value (97.5% in the following numerical example). No capital allocation at times following $t = 0$ is allowed for. Hence, the insurer must allocate assets at time $t = 0$ only. These assets are partially funded by the single premium paid for the annuity purchase, while the remaining part consists of own capital.

Figure 4.4, with respect to the size of the portfolio, illustrates the behaviour of the (relative) required solvency margin, defined as

$$\frac{\text{required assets - portfolio reserve}}{\text{portfolio reserve}}$$

the two quantities being referred to time $t = 0$. Evaluations have been performed according to the deterministic approach using the scenario [med], and the probabilistic approach weighting the three scenarios with the probabilities stated above.

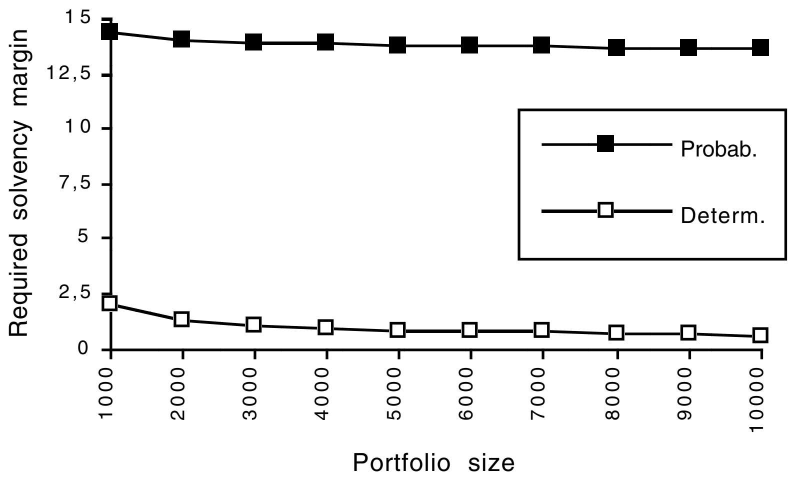


FIGURE 4.4 - *Required solvency margin* ($\times$ *100)*

The deterministic approach only allows for the random fluctuation risk, which can be hedged by increasing the portfolio size. Actually, as shown by Figure 4.4, the required solvency margin tends to zero as the portfolio size increases. On the contrary, the probabilistic approach clearly reveals the presence of the longevity risk, resulting in a much higher solvency requirement. Moreover, the solvency requirement has a (high) positive asymptotic value witnessing the systematic component of the mortality risk, i.e. the longevity risk.

In more general terms, the longevity risk can be faced by:

(i) a safety loading of premiums (possibly with an annuity adjustment mechanism aiming at distribution of mortality profits to the annuitants);

(ii) an adequate solvency margin (or risk-based capital);

(iii) reinsurance arrangements.

Some aspects concerning point (i) are dealt with in Section 5.2. Here we focus our attention on some reinsurance arrangements facing the longevity risk.

Various reinsurance arrangements can be conceived, at least in principle. In particular:

(1) a surplus reinsurance, aiming at ceding part of high amount annuities;

(2) an XL-like reinsurance treaty, to be structured in such a way that the reinsurer pays the final part of the annuity while exceeding a given term (for example, while exceeding the age of 85; see Figure 4.5);

(3) a stop-loss reinsurance, aiming at partially covering the required portfolio reserve (see Figure 4.6).

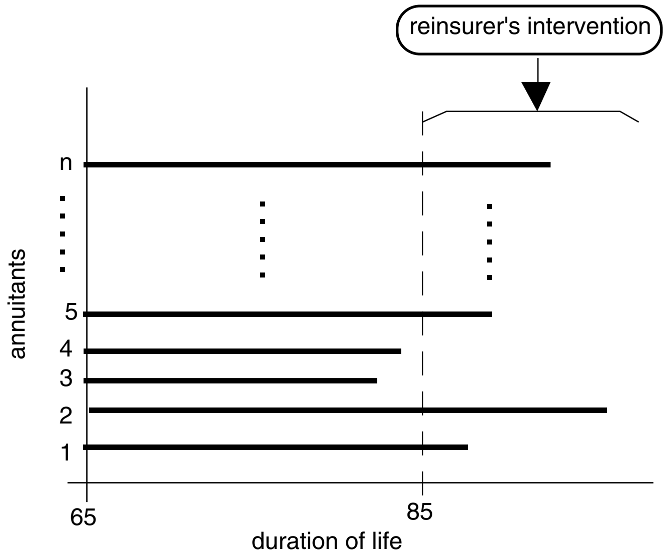


FIGURE 4.5 - *An XL reinsurance arrangement*

However, in realistic terms, reinsurance arrangements defined on a short-medium period basis should be addressed, in order to avoid too high safety loadings in the reinsurance premiums. To this purpose, stop-loss arrangements could provide interesting reinsurance coverages. According to the stop-loss logic, reinsurer's interventions aim at preventing the unsolvency of the cedant, caused by mortality deviations (systematic as well as random). The effect of mortality deviations can be perceived, for example, comparing the assets available at a given time with the portfolio reserve required to meet the insurer's obligations related to the annuitants surviving at that time (see Figure 4.6). Hence, the reinsurer's intervention can be based on this comparison. A too short period can emphasize the random deviation effects; conversely, a too long period implies a severe longevity risk for the reinsurer, and then (as mentioned above) very high reinsurance premiums.

Consistently with a stop-loss reinsurance arrangement, the assessment of the longevity risk should be based on the evaluation of the probability distributions of "losses" meant as differences between the required portfolio reserve and the available assets. When simple portfolios are concerned (tipically one-cohort portfolios) these probability distributions can be easily derived from the probability distributions of the random number of survivors. A reinsurance arrangement of the type described above was proposed by Marocco and Pitacco (1998), which the reader is referred to for some numerical examples, built up with both analytical and simulation methods.

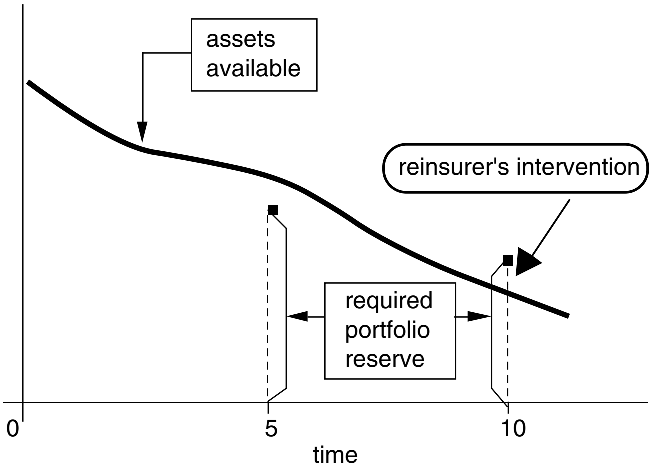


FIGURE 4.6 - *A Stop-Loss reinsurance arrangement*

### 4.3 LTC insurance

For the sake of brevity, we focus our attention on a particular LTC product only, the so-called enhanced pension. This product provides a straight life annuity from the retirement age on, uplifted in case the annuitant becomes disabled (according to a given definition of LTC disability).

The following hypotheses are adopted. Only one level of disability is considered. Because of the usually chronic character of LTC disability, the possibility of recovery from the LTC state is disregarded. Randomness other than the demographic one (coming from mortality and disability) is disregarded; in particular no investment risk is taken into account.

In order to appraise the risk inherent in a tariff structure for LTC covers uncertainty of the future evolution of both mortality and senescent disability must be modelled. Following Ferri, Olivieri (2000) and Olivieri, Pitacco (2001a), different scenarios have been focussed, each one including a given projection of mortality and disability trends. The scenarios have been defined in terms of the evolution of the expected time spent in the healthy state and in the disability state.

To this purpose the following quantities, referred to a person with a given age, must be considered:

$e_{H,H}$ = expected time spent in the healthy state by a healthy person, i.e. healthy life expectancy for a healthy person;

$e_{H,LTC}$ = expected time spent in the LTC state by a healthy person, i.e. disability life expectancy for a healthy person;

$e_{LTC}$ = life expectancy for a disabled person;

$e_H = e_{H,H} + e_{H,LTC}$ = total life expectancy for a healthy person.

With reference to a person entering insurance in the healthy state, the three theories described in Section 3.1 can be expressed in terms of the evolution of life expectancies as follows:

(i) when compared to past data, $e_H$ increases with a major contribution (in relative terms) from $e_{H,H}$;

(ii) $e_H$ increases with a major contribution (in relative terms) from $e_{H,LTC}$;

(iii) $e_{H,H}$ and $e_{H,LTC}$ increase at similar rates.

To express different scenarios, analytical laws for mortality and disability rates have been used in performing numerical evaluations. Mortality has been represented by a Weibull law, while disability inception rates have been modelled using a Gompertz law.

A scenario space consisting of five scenarios, $\{S_1, S_2, S_3, S_4, S_5\}$, has been considered; the five items consist in combined hypotheses about mortality and disability trends, expressed by the parameters of the analytical laws. Scenario $S_C$, coming from recent cross-sectional observations of mortality and disability of elderly people, has been taken as a starting point for the construction of the five projected scenarios. Table 4.1 shows the values of the expectancies $e_{H,H}$, $e_{H,LTC}$, $e_H$, $e_{LTC}$ under each scenario.

| Scenario | Life expectancy for Healthy people | | | Life expectancy for LTC people |
|---|---|---|---|---|
| | in the Healthy state $e_{H,H}$ | in the LTC state $e_{H,LTC}$ | total $e_H$ | $e_{LTC}$ |
| $S_C$ | 14.428 | 1.566 | 15.995 | 15.307 |
| $S_1$ | 15.156 | 1.435 | 16.591 | 15.931 |
| $S_2$ | 16.042 | 1.563 | 17.605 | 16.983 |
| $S_3$ | 15.844 | 1.749 | 17.593 | 16.983 |
| $S_4$ | 15.501 | 2.073 | 17.574 | 16.983 |
| $S_5$ | 16.577 | 2.366 | 18.943 | 18.397 |

TABLE 4.1 - *Life expectancies*

In the scenario construction, it has been assumed that the total life expectancy $e_H$ will increase anyhow, as suggested by mortality trends in populations including both healthy and disabled people. As far as contributions from $e_{H,H}$ and $e_{H,LTC}$ are concerned, it is easy to deduce, comparing the results in Table 4.1, that scenario $S_1$ implies lower expected costs than the others, representing consequences depicted by the compression theory. On the other side, $S_5$ is the scenario with the highest expected costs, corresponding to the consequences of the pandemic theory. Scenario $S_3$ is, in some sense, intermediate between the above depicted scenarios, reflecting the equilibrium theory. Finally, $S_2$ and $S_4$ depict projections that are intermediate between scenario $S_3$ and the two "extreme" scenarios.

The riskiness of the enhanced pensions has been evaluated according to a deterministic approach and a stochastic one as well (see Section 4.1). Following Olivieri and Pitacco (2001a), here we analyse the riskiness in terms of solvency requirements. As far as the definition of solvency requirements is concerned, the approach described in Section 4.2 is adopted.

Figure 4.7 illustrates the behaviour of the required solvency margin. Note that small portfolio sizes have been addressed because of the type of insurance cover, actually less common than life annuity products. Again, the deterministic approach simply reveals the hedging opportunity provided by the portfolio size. On the contrary, the probabilistic approach singles out the presence of the systematic component of the demographic risk, witnessed by the high asymptotic value of the required solvency margin.

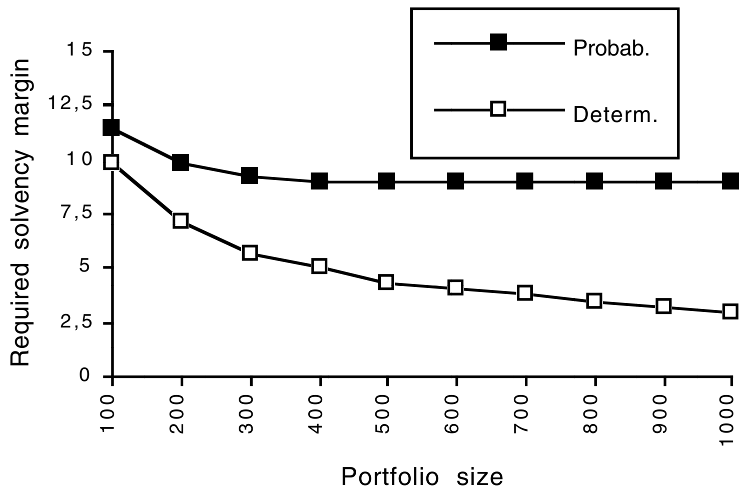


FIGURE 4.7 - *Required solvency margin (× 100)*

Reinsurance arrangements can help in facing the longevity risk in an LTC portfolio. Let us focus on a stop-loss reinsurance arrangement as described in Section 4.3, and assume that the reinsurer intervenes when the assets are less than 90% of the required portfolio reserve.

In Figure 4.8 the solvency requirement in the presence of a stop-loss treaty is compared with the solvency requirement when no reinsurance works; results have been obtained adopting the probabilistic approach. It is worth noting that in the presence of a reinsurance treaty, the required solvency margin decreases as the portfolio size increases (within the range considered); this fact witnesses that a significant portion of the systematic risk moves from the cedant to the reinsurer. For more details the reader can refer to Olivieri and Pitacco (2001a).

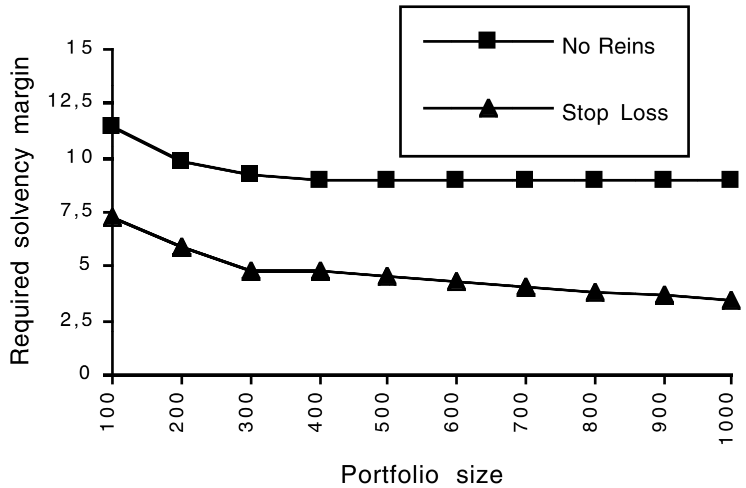


FIGURE 4.8 - *Required solvency margin (× 100) with a reinsurance arrangement*

## 4.4 Sickness benefits for the elderly

In this Section sickness covers providing medical expense reimbursement are addressed. In particular we focus our attention on lifetime post-retirement sickness covers.

Uncertainty affecting lifetime sickness insurance originates from various causes. The following classification gives an insight into the randomness of a lifetime sickness cover, and can help in appreciating the role of a premium system in determining the randomness itself. Uncertainty comes from:

(a) the random number of claim events in any given insured period (in particular, in each year);

(b) the random amount (medical expenses refunded) relating to each claim;

(c) the random duration of the life of the insured.

Note that items (a) and (b) are common to all covers in the framework of general insurance. The relevant effects can be faced by adopting appropriate premium calculation principles, and in particular by charging premiums with a convenient safety loading. However, it should be stressed that paucity of sickness data relating to very old ages increases the difficulty in assessing the randomness originated from items (a) and (b). Item (c) represents the mortality risk; its impact is strongly related with the premium system adopted.

Several premium systems can be conceived and actually used in pricing post-retirement sickness covers. In particular:

(1) a single premium at retirement age only, fully meeting the future costs of the insurer;

(2) a sequence of level premiums, determined according to a given premium calculation principle;

(3) a sequence of "natural" premiums, each premium meeting the costs of the relevant year; in this case, benefits are funded on a "pay as you go" basis;

(4) mixtures of (1) and (2), i.e. a single premium (partially meeting the future costs) plus a sequence of level premiums;

(5) mixtures of (1) and (3), i.e. a single premium (partially meeting the future costs) plus a sequence of annual premiums proportional to natural premiums (for instance, a given percentage of the natural premiums).

Each premium system leads to a different reserve accumulation process (and this implies a different exposure to the longevity risk, as mentioned below). In particular, the premium system (1) requires that the whole single premium is initially reserved, while the annual expected costs are progressively funded drawing from the reserve (see Figure 4.9). The premium system (2) leads to a reserve initially increasing and finally decreasing, because of the behaviour of annual expected costs (see Figure 4.10). The premium system (3) does not require, by definition, any reserve accumulation.

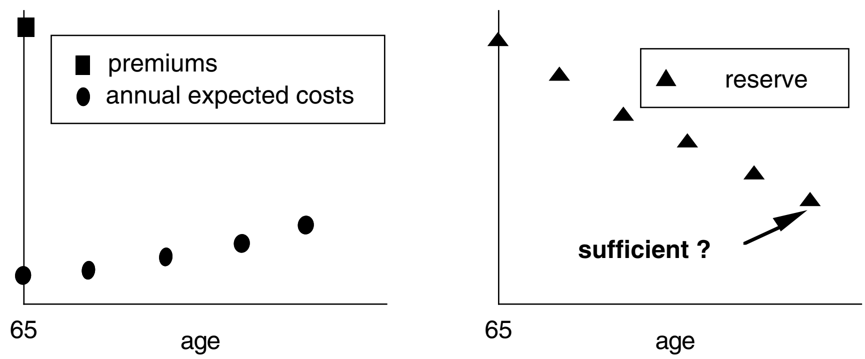


FIGURE 4.9 - *Funding sickness benefits: the case of a single premium*

Each premium arrangement has interesting features from the point of view of the insurer or from the point of view of the insured. For example, a single premium (1) entirely meeting the future costs may be of great interest for the insured if a lump sum (e.g. a survival benefit provided by an endowment policy) becomes available at the retirement age. Conversely, a sequence of natural premiums (3) can intuitively reduce the variability of the portfolio loss and hence can be appealing for the insurer. Mixtures (4) and (5) may constitute good compromises. For example, a mixture (4) is commonly used for funding costs related to Continuous Care Retirement Communities in the U.S.; in this case an advance fee (a single premium) is followed by a sequence of periodic fees (periodic premiums), possibly adjusted for inflation.

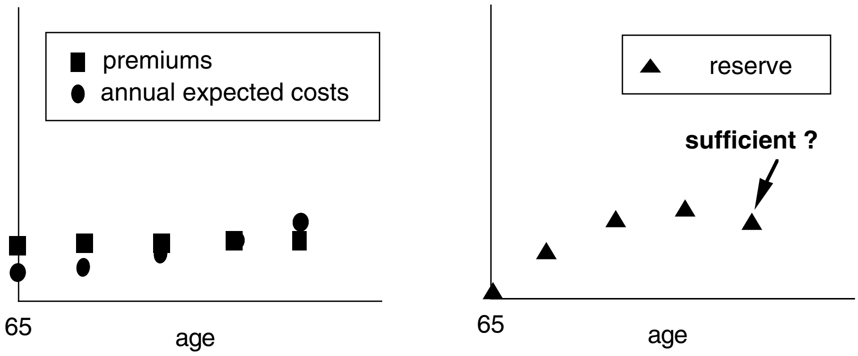


FIGURE 4.10 - *Funding sickness benefits: the case of lifetime level premiums*

The problem of funding lifetime post-retirement sickness covers, in the presence of longevity risk, is analysed by Olivieri and Pitacco (1999). The paper focusses on the mortality risk (and in particular on its longevity component), disregarding the other sources of risk, and aiming to assess the longevity risk as a function of the premium system adopted. For a deep analysis of longevity risk assessment in post-retirement sickness benefits the reader can refer to this paper. Now we only address a particular problem, which can be of great practical interest.

Assume that, for any given premium system, the riskiness of a post-retirement sickness cover portfolio is quantified by the variance of the random loss of the portfolio itself, i.e. the random present value of future benefits less the random present value of future premiums.

Let $E$ denote the expected present value of future benefits for a given policy. The amount of the single premium paid at time $0$, $\Pi(\alpha)$, can be expressed in terms of $E$:

$$\Pi(\alpha) = \alpha E ;$$

the annual premiums can be denoted by $\pi_k(\alpha)$, $k = 1, 2, ...,$ and in particular by $\pi(\alpha)$ if constant.

If $\alpha = 1$ we have the case in which a single premium is only paid (case (1)); if $\alpha = 0$ we find the cases in which level premiums (case (2)) or natural premiums (case (3)) will be paid; finally, if $0 < \alpha < 1$ a premium paid in 0 will be followed by a sequence of annual premiums (mixtures (4) and (5)). Note that letting $0 \le \alpha \le 1$ all the premium systems can be expressed in terms of the mixtures (4) and (5).

For any given premium system, the variance of the loss function increases as the parameter $\alpha$ increases, because of the higher portion of benefits financed by the single premium paid in 0 and hence because of the higher level of pre-funding, which can reveal itself not sufficient to meet future costs if the actual mortality improvement is higher than the expected one.

Moreover, for any value of $\alpha$ (with the obvious exception of $\alpha = 1$) the variance is smaller in premium systems of type (5) than in premium systems of type (4), which means that annual premiums proportional to annual costs (i.e. proportional to natural premiums) are less risky than level premiums, because of the lower reserve levels in the former case.

However, from a commercial point of view, level premiums $\pi(\alpha)$ are usually preferred. In order to design appealing premium arrangements, but aiming at limiting risk, the insurer may adopt level premiums charged with an appropriate loading.

Assume a proportional loading, so that the charged level premium is given by

$$\pi(\alpha; \lambda) = (1 + \lambda)\, \pi(\alpha) .$$

Properly choosing the loading parameter $\lambda$, the insurer can reduce the variance of the loss function for a premium system of type (5), possibly obtaining the variance of the corresponding system of type (4).

Table 4.2 illustrates some values for the parameter $\lambda$ as a function of the parameter $\alpha$ which defines the premium arrangement. The number of policies is denoted by $N$. Results are drawn from the paper by Olivieri and Pitacco (1999), in which three future mortality scenarios are considered and weighted as described in Section 4.1.

Note that, for any given value of $\alpha$, the requested loading decreases as the portfolio size increases. However, the asymptotic value is greater than 0, and this is clearly due to the presence of the systematic component of the mortality risk, i.e. the longevity risk.

Finally, it is worth stressing that the premium loading structure considered is not meant as a tool for facing longevity risk. The aim is simply to arrange premiums so that the riskiness (in terms of variance of the loss funcion) is the lowest for a given initial payment.

| $\alpha$ | N = 100 | N = 1000 | N = 10000 | N = 100000 |
|---|---|---|---|---|
| 0.2 | 0.2470 | 0.1933 | 0.1823 | 0.1811 |
| 0.4 | 0.2254 | 0.1891 | 0.1814 | 0.1805 |
| 0.6 | 0.2191 | 0.1877 | 0.1810 | 0.1803 |
| 0.8 | 0.2160 | 0.1871 | 0.1809 | 0.1802 |
| 1 | 0 | 0 | 0 | 0 |

TABLE 4.2 - *Loading parameter* $\lambda$

### 4.5 Packaging benefits to face the longevity risk

Pension schemes usually provide combinations of life (and possibly health) insurance benefits arranged for a given group of persons, for example the employees of a company.

For brevity, let us consider benefits depending on the duration of life only, viz. old age pensions (life annuities), lump sum death benefits, survivor pensions and reversionary pensions. Hence, only mortality risks are involved.

Using various quantities to assess the effect of the mortality risks (e.g. variance, risk index and some percentiles of the distribution of the present value of pension scheme liabilities), it is possible to prove that the combination of benefits plays an important role in reducing the impact of these risks (at the same time providing the customers with a wider range of covers).

Interesting results, with numerical illustrations, can be found in Biffis and Olivieri (2002). Here we simply point out the following feature. Although combining several types of benefits can increase the overall risk, possibly because of positive correlations, in relative terms the risk decreases. This means in particular that the ratio between the amount of money required for solvency purpose and the mathematical reserve is lower when more benefits are provided. In other words, a better hedging of the mortality risk can rely on a larger overall mathematical reserve.

## 5 Generalizing the structure of life annuities

The present Section aims at stimulating the debate about life annuities, meant as an important tool to provide post-retirement income. Annuity products will be addressed in the context of overall post-retirement income planning. Of course, attention will be mostly devoted to demographic issues, to keep the presentation in line with the main scope of this paper. Nevertheless, important financial aspects should not be disregarded when assessing and comparing the various opportunities meeting the post-retirement income needs.

Here we disregard the distinction between "pension annuities" (i.e. annuities linked in some way with occupational pension schemes) and "purchased life annuities" (i.e. annuities based on an individual policy as a result of a strictly personal choice, not involving pension scheme rules). Actually, the distinction varies from country to country, also depending on specific tax rules.

Innovative ideas and proposals are presented and discussed in the reports by the British Department for Work and Pensions (2002) and the Retirement Choice Working Party (2001). The paper by Wadsworth and Findlater (2002) suggests a technical structure for a fund providing annuities. A comprehensive description of several annuities markets is provided by Cardinale, Findlater and Orszag (2002). Various arrangements for annuities in presence of the longevity risk are dealt with, in their technical perspective, by Olivieri and Pitacco

(2001b). Milevsky and Promislow (2001) analyse mortality guarantees involved by the option to annuitise.

## 5.1 Flexibility in financing post-retirement income

We now propose a framework which can aid discussion and assessment of new ideas in the area of annuity design. We assume that an accumulation process takes place throughout the working period of an individual. After retirement, a decumulation process takes place and hence income requirements are met using, in some way, the accumulated fund.

Figure 5.1 illustrates the process consisting in:

(1) accumulation of contributions during the working period;

(2) (possible) annuitisation of (part of) the accumulated fund (before or after retirement);

(3) getting post-retirement income from life annuities or through income drawdown.

The annuitisation of (part of) the accumulated fund consists in purchasing a deferred annuity if annuitisation takes place during the accumulation period, an immediate annuity otherwise. Hence, at any time resources available for financing post-retirement income are shared between a non-annuitised fund and an annuitised one. It is sensible to assume that a higher degree of flexibility in selecting investment opportunities concerns the non-annuitised fund.

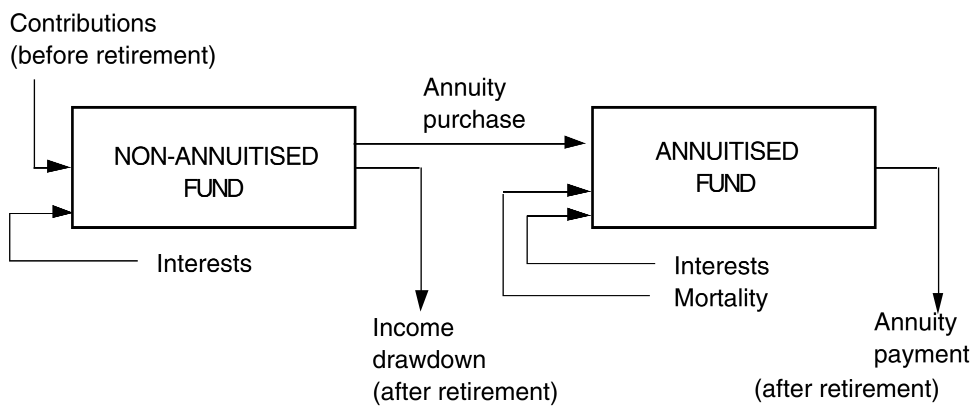


FIGURE 5.1 - *From accumulation to post-retirement income*

Figures 5.2 and 5.3 illustrate the behaviour of the non-annuitised fund and the annuitised fund respectively. Effects of the annuity purchase (jumps in the processes), of income drawdown and of annuity payment are singled-out.

The slope of the non-annuitised fund depends, while the fund itself is increasing, on both contributions and interests, whereas it depends on the drawdown policy while the fund is decreasing. As regards the annuitised fund, its slope depends on interests and mortality (reserves pertaining to people dying are assumed to be credited to the survivors) while it is increasing, whereas it depends on annuity payment while decreasing.

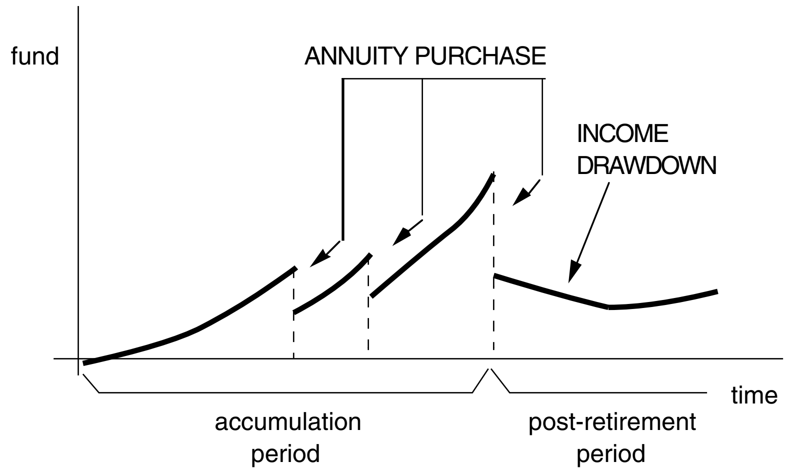


FIGURE 5.2 - *The non-annuitised fund*

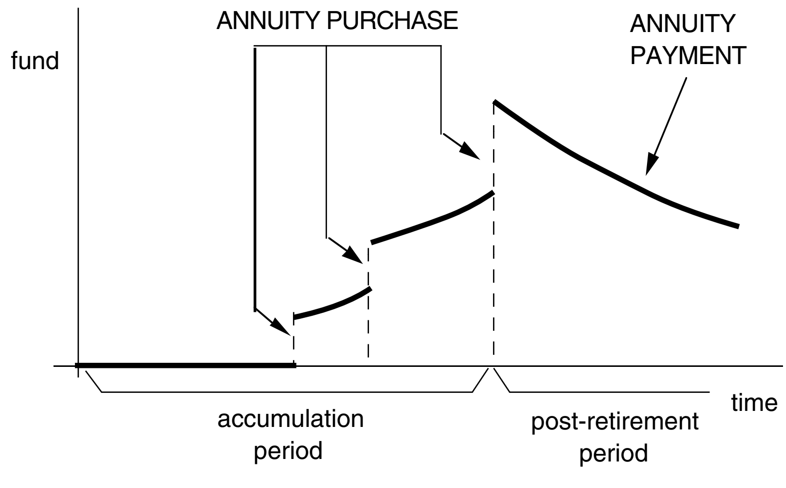


FIGURE 5.3 - *The annuitised fund*

Let denote with $F_{NA}(t)$ and $F_A(t)$ the values of the non-annuitised fund and the annuitised fund respectively, at time $t$. The degree of the annuitisation policy can be summarized by the annuitisation ratio $R(t)$, defined as follows:

$$R(t) = \frac{F_A(t)}{F_{NA}(t) + F_A(t)} .$$

Note that, obviously, $0 \le R(t) \le 1$, and that $R(t) = 0$ means that, up to time $t$, no annuity has been purchased, whilst $R(t) = 1$ means that, at time $t$, the whole fund available consists in reserves related to purchased annuities.

Figures 5.4 to 5.7 illustrate some strategies for financing post-retirement income. In most cases, the technical tool provided by the life annuity is involved. The various strategies are described in terms of the annuitisation ratio profile, thus the value of $R(t)$ is plotted against time $t$.

To make interpretation easier, initially let us suppose that a specified mortality assumption is adopted when annuitising (a part of) the accumulated fund and that the assumption itself cannot be replaced in relation to the purchased annuity, whatever the mortality trend might be.

Figure 5.4 illustrates two "extreme" choices. The choice (1) consists in building up a traditional deferred annuity. In this case, each amount paid to the accumulation fund (possibly a level premium, or a single recurrent premium) is immediately converted into a deferred annuity; hence the accumulated fund is completely annuitised. Post-retirement income requirements are met by the annuity (a flat annuity or, possibly, a rising profile annuity, viz an escalating annuity or an inflation-linked annuity).

The choice (2) represents the opposite extreme. No annuitisation works, whence income requirements are fulfilled by income drawdown, which implies spreading the fund accumulated at retirement, over future life expectation, according to some spreading rule. Sometimes pensioners prefer this choice because of the high degree of freedom in selecting investment opportunities even during the post-retirement period.

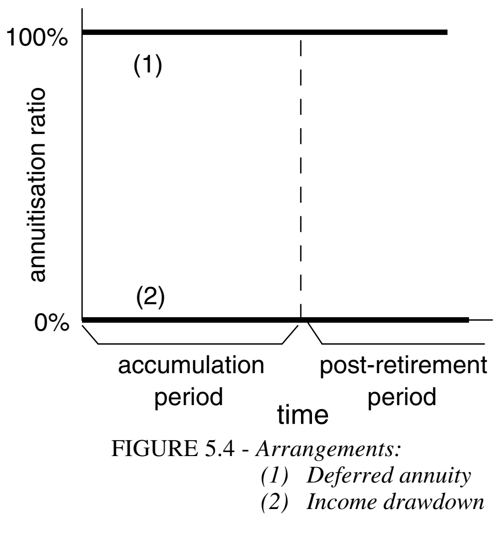

FIGURE 5.4 - *Arrangements:*
*(1) Deferred annuity*
*(2) Income drawdown*

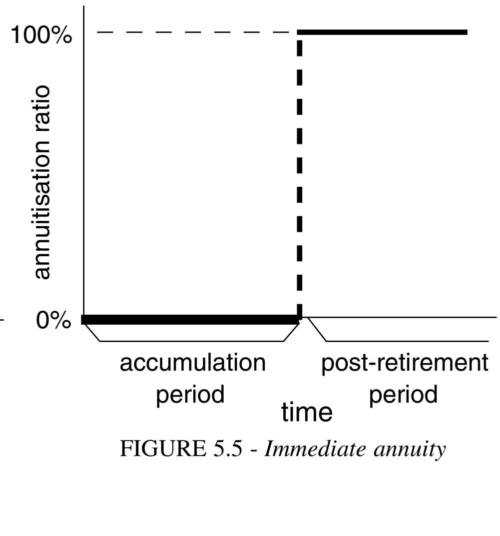

FIGURE 5.5 - *Immediate annuity*

It should be stressed that choice (1) leads to an inflexible post-retirement income, whilst choice (2) allows the pensioner to adopt a spreading rule consistent with a specific income profile. Conversely, it is worth noting that arrangement (1) completely transfers the mortality risk (including its longevity component) to the annuity provider, whilst according to arrangement (2) the mortality risk is suffered by the pensioner only.

In more general terms, the process of mortality risk transfer depends on the annuitisation profile: the portion of mortality risk transferred from the pensioner to the annuity provider increases as the annuitisation ratio increases. The following arrangements constitute practical examples of how mortality risk can be transferred, as time goes on, to the annuity provider.

The annuitisation of the fund at retirement date only is illustrated in Figure 5.5, which depicts the particular case of a complete annuitisation. This arrangement is characterized by flexibility in the investment choice during the accumulation period; conversely it produces an inflexible post-retirement income profile.

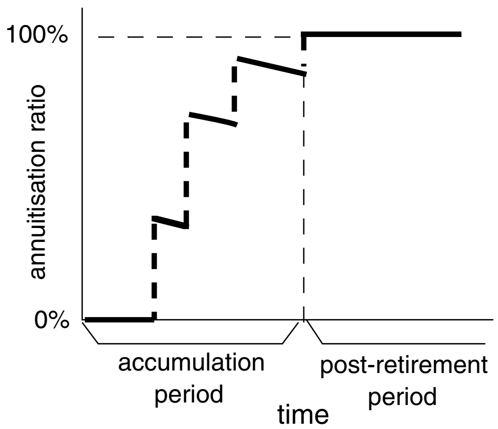


FIGURE 5.6 - *Combined annuities*

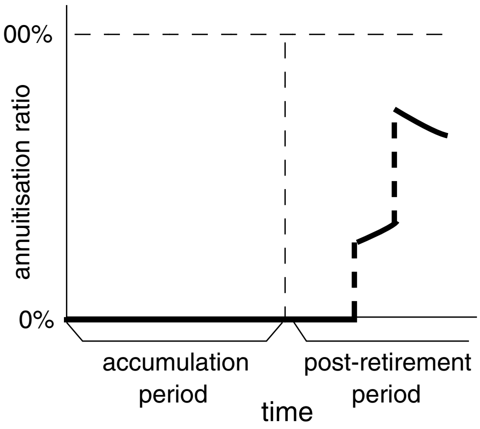


FIGURE 5.7 - *Staggered annuitisation*

In Figure 5.6 the annuitisation ratio increases during the accumulation period because of the positive jumps corresponding to the purchase of annuities with various deferment periods (which can constitute, in some contracts, the exercise of an option to annuitise). The behaviour of the annuitisation ratio between jumps obviously depends on contribution and interests affecting the non-annuitised fund as well as on interests and mortality as regards the annuitised fund.

Conversely, Figure 5.7 illustrates the case in which no annuitisation is made throughout the accumulation period, whereas the fund available after the retirement date is partially used to purchase annuities; such a process is sometimes called "staggered annuitisation" or "staggered vesting". The behaviour of the ratio between jumps depends on interests and income drawdown as regards the non-annuitised fund as well as interests and mortality as regards the annuitised fund.

Arrangements like those illustrated by Figures 5.6 and 5.7 are characterized by a high degree of flexibility as regards both the post-retirement income profile and the choice of investment opportunities for the non-annuitised fund.

The framework proposed above clearly shows the wide range of choices consisting in different annuitisation strategies. So, convenient investment and annuity products can be designed, complying with different needs and preferences of the clients.

## 5.2 Mortality guarantees

Sofar we have supposed that a specified mortality assumption is adopted when (a part of) the available fund is annuitised and that, as regards the purchased annuity, the assumption itself cannot be replaced, whatever the mortality trend might be. In particular when a deferred annuity purchased during the accumulation period is concerned, this assumption leads to a very high level of mortality guarantee embedded in the annuity product, and, because of the systematic component of the mortality risk, this implies huge financial requirements in order to face the longevity risk (see Section 4.2).

The guarantee degree can be kept at more reasonable levels if the mortality assumption is allowed to vary throughout the deferment period, if significant deviations from the assumed mortality trend are experienced.

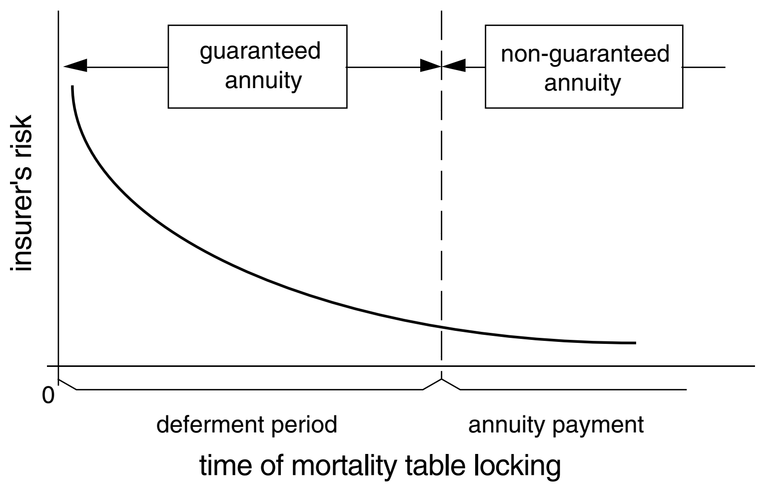


FIGURE 5.8 - *Mortality guarantee in an annuity product*

It is worth noting that if the mortality assumption may be adjusted during the deferment period only, the amount of the annuity is ultimately determined at the end of this period, and the amount itself is then guaranteed for the whole lifetime. On the contrary, if the mortality assumption is allowed to vary even during the payment period, the resulting product is a (rather poor value) non-guaranteed annuity.

Obviously the location of the mortality risk (and in particular of its longevity component) varies according to the time at which the mortality assumption is locked (see Figure 5.8). In particular, the pensioner suffers a large part of the longevity risk if the mortality assumption may be adjusted at any time even during the annuity payment period.

In any case, a larger retention of longevity risk by the annuity provider should be financed by a higher premium for the annuity product. Actually, an appropriate safety loading should be calculated and applied, as the result of pricing the longevity guarantee embedded in the annuity structure. The higher premium could be justified in terms of a greater value of the annuity product. Obviously, further value can be added to the product by adopting some annuity adjustment mechanism aiming at the distribution of mortality profits to annuitants.

# 6 Conclusions

The present paper mainly aims at providing a (rather informal) introduction to the longevity risk, arising from the uncertainty in future demographical trends, and the financial requirements following the impact of the longevity risk itself on insurance covers involving living benefits (life annuities, LTC benefits, whole life sickness benefits, etc.).

From some numerical examples, the dramatic importance of the uncertainty in future demographical trends clearly emerges. In our opinion, great attention should be devoted especially in analysing possible integrations between reinsurance arrangements and capital allocation policies, in particular from a corporate point of view.

Probably, a deeper knowledge of mortality and disability trends and more actuarial research specifically dealing with this particular risk can help, in a near future, in establishing a unified approach to reinsurance arrangements, safety loadings, solvency margin requirements.

Of course, actuarial aspects must be carefully considered while designing new insurance products involving living benefits. Life annuity products provide an important example of the need for new ideas consistent with a rapidly evolving demographic scenario.

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